J. Fluid Mech. (1969), *vol.* 35, *part* 2, *pp.* 299–310
Printed in Great Britain

Acoustic energy flux from shock-turbulence interaction

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(Received 9 October 1967 and in revised form 12 July 1968)

The analysis of the sound field generated by the passage of isotropic turbulence through a shock of finite strength (Ribner 1953, 1954) has been extended to provide the flux of acoustic energy emanating from unit area on the downstream side of the shock. This is motivated by the problem of estimating the sound power emerging from a supersonic jet containing shock waves. The energy flux is found to vary almost linearly with shock density ratio, reaching a maximum at infinite Mach number of 0·062 of the flux of turbulence kinetic energy convected into unit area of the shock.

Direct comparison with a result obtained by Lighthill (1953) is misleading. His energy relations, reckoned relative to a frame moving with the fluid, must be converted to the shock-fixed frame used herein. The converted results of his theory (weak shocks) and the results of our theory (arbitrary shocks) appear to show a similar asymptotic behaviour for vanishing shock strength; they diverge with increasing shock strength.

1. Introduction

It is known that the passage of turbulence through a shock wave generates relatively strong sound waves. The sound-pressure level has been evaluated (Ribner 1953, 1954) for an idealized model involving homogeneous turbulence convected through an infinite plane shock. This has only limited application, however, to such matters as the shock-turbulence interaction in a supersonic jet: local sound pressures within the jet and nearby may be predicted roughly, but not far-field effects nor total power.

The flux of acoustic energy from unit area of the shock, on the other hand, would be relevant to power estimation. Calculations of this flux have been carried out only for an acoustical model of a shock (Lighthill 1953). In the present paper the analysis for finite shock strength (Ribner 1954) is extended to provide the acoustic energy flux.

2. Shock-turbulence interaction

According to the Fourier integral theorem a turbulent velocity field can be represented as a superposition or spectrum of elementary waves distributed among all orientations and wavelengths. The waves are transverse for weak turbulence because of the constraint of incompressibility (even though convected at high speed). Thus a single spectrum wave can be interpreted physically

(Ribner & Tucker 1952; Batchelor 1953) as a plane sinusoidal wave of shearing motion (figure 1).

When a turbulence pattern is convected into a shock wave, the individual shear waves are abruptly altered on passing through. In addition, the shock

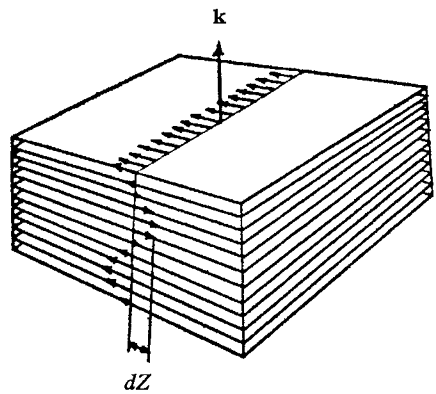

FIGURE 1. Single Fourier component of a weak turbulent flow (shear wave).

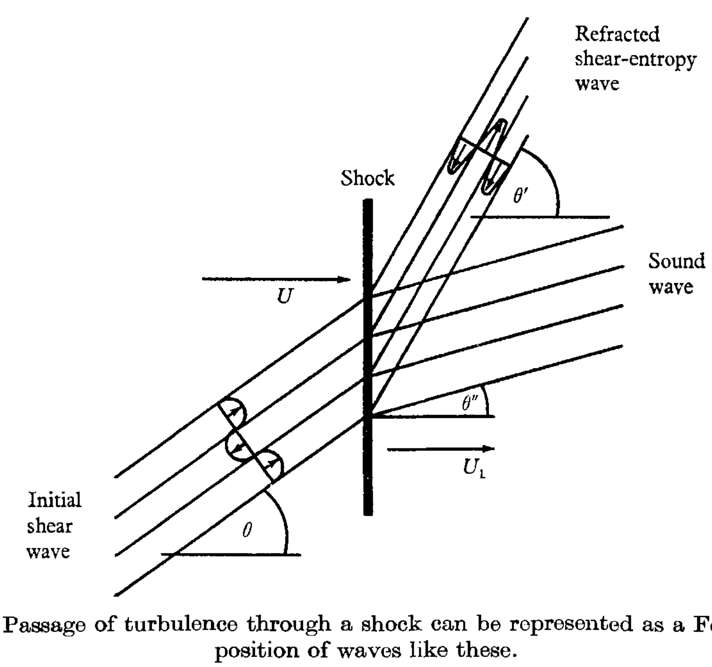

FIGURE 2. Passage of turbulence through a shock can be represented as a Fourier superposition of waves like these.

interaction generates a pattern of sound waves (and entropy waves) on the downstream side. If the initial turbulence wave pattern is known statistically then the sound-wave pattern can be determined statistically. That is, spectra, correlations, and mean square values can be calculated.

The shock interaction for an arbitrary shear wave encountering an infinite plane shock is shown in figure 2. The quantitative relations were worked out and incorporated into a statistical analysis for homogeneous turbulence by Ribner

(1953, 1954). A brief account of relevant parts of the development is given below. The key quantities are non-dimensional: u is longitudinal velocity component/ a^* (= critical sound speed); p'' is perturbation pressure/p_1 (= ambient pressure).

We may express the velocity field of an individual shear wave as (figure 1),

$$du = dZ_u(\mathbf{k}) \exp(i\mathbf{k}.\mathbf{x}), \tag{1}$$

with similar expressions for dv and dw. The integral of du, dv, dw over $\mathbf{k}$-space is then the Fourier–Stieltjes integral for the turbulent velocity field.

In (1) du is the velocity component normal to the shock, and the planes of constant phase $\mathbf{k}.\mathbf{x}$ = constant make an angle θ with this normal (figures 1, 2). The pressure dp'' in the sound wave that arises from the shock interaction is

$$dp'' = dZ_{p''}(\mathbf{k}'') \exp(i\mathbf{k}''.\mathbf{x}), \tag{2}$$

where the phase planes $\mathbf{k}''.\mathbf{x}$ = constant are inclined at angle θ'' to the shock normal.

The amplitudes of the two waves are related by

$$dZ_{p''} = P(\theta)\, dZ_u, \tag{3}$$

where the transfer function $P(\theta)$ is the result of a gasdynamic calculation (tabulated in Ribner (1954)). The dependence of P solely on the angle θ is to be noted.

It is necessary to invoke statistical relations to deal with the random nature of the turbulent field and of the sound field it produces. If we form the ensemble average for two waves of different wave-numbers $\mathbf{k}$ and $\boldsymbol{\kappa}$, it is easily proved that

$$\overline{dZ_u(\mathbf{k})\, dZ_u^*(\boldsymbol{\kappa})} = \delta(\boldsymbol{\kappa}-\mathbf{k})[uu]\, d^3\mathbf{k}\, d^3\boldsymbol{\kappa} \tag{4}$$

according to Tatarski (1961). Here $[uu]$ is our special symbol for the spectral density of $\langle u^2 \rangle$ in wave-number space $\mathbf{k}$. The vanishing of $\delta(\boldsymbol{\kappa}-\mathbf{k})$ for $\boldsymbol{\kappa} \neq \mathbf{k}$ implies that waves of different wavelengths or inclinations (since $\boldsymbol{\kappa}$ and $\mathbf{k}$ are vectors) are statistically independent.

The integral of (4) over $\boldsymbol{\kappa}$-space may be written, by virtue of the δ-function,

$$\overline{dZ_u(\mathbf{k})\, dZ_u^*(\mathbf{k})} = [uu]\, d^3\mathbf{k} \qquad \text{velocity (shear wave).} \tag{5}$$

Similarly,

$$\overline{dZ_{p''}(\mathbf{k}'')\, dZ_{p''}^*(\mathbf{k}'')} = [p''p'']\, d^3\mathbf{k}'' \qquad \text{pressure (sound wave).} \tag{6}$$

The integral of (5) over $\mathbf{k}$-space is $\langle u^2 \rangle$ and the integral of (6) over $\mathbf{k}''$-space is $\langle p''^2 \rangle$.

From (3), (5) and (6)

$$\langle p''^2 \rangle = \int_\infty |P(\theta)|^2 [uu]\, d^3\mathbf{k}. \tag{7}$$

The initial turbulence is now restricted to be isotropic, so that its longitudinal spectral density has the general form (Batchelor 1953)

$$[uu] = k^2 F(k) \cos^2\theta \tag{8}$$

in spherical polar co-ordinates in wave-number space. In these co-ordinates

$$d^3\mathbf{k} = k^2 \cos^2\theta\, dk\, d\phi\, d\theta, \tag{9}$$

where the azimuth ϕ is the angle between the projection of $\mathbf{k}$ on the shock plane and some reference line on the shock. Thus, by (5)

$$\langle u^2 \rangle = 2\int_0^\infty k^4 F(k)\,dk \int_0^{2\pi} d\phi \int_0^{\frac{1}{2}\pi} \cos^3\theta\, d\theta. \tag{10}$$

It follows from (7) to (10) that the mean square pressure in the sound pattern $\langle p''^2 \rangle$ is related to the mean square longitudinal velocity $\langle u^2 \rangle$ in the initial turbulence by

$$\langle p''^2 \rangle = \tfrac{3}{2}\langle u^2 \rangle \int_0^{\frac{1}{2}\pi} |P(\theta)|^2 \cos^3\theta\, d\theta. \tag{11}$$

The contributions to $\langle p''^2 \rangle$ from waves at each angle θ'' are statistically independent (cf. remarks after (4)). Thus the contribution associated with sound waves at angle θ'' in the range $d\theta''$ is

$$d\langle p''^2(\theta'')\rangle = \tfrac{3}{2}\langle u^2 \rangle |P(\theta)|^2 \cos^3\theta\, d\theta, \tag{12}$$

where $d\theta''$ is known in terms of $d\theta$.

Equations (11) and (12) cover two classes of sound waves associated with respective ranges of the inclinations θ'' and θ. For $\theta_{\mathrm{cr}} \leqslant |\theta| \leqslant \frac{1}{2}\pi$, the wave amplitude $|P(\theta)|$ is constant with distance x downstream of the shock. For the smaller inclinations $0 \leqslant \theta \leqslant \theta_{\mathrm{cr}}$ the amplitude $|P(\theta)|$ decays exponentially, so that these waves constitute the 'near-field'. These near-field waves carry no acoustic energy—the pressure and particle velocity are out of phase—and are not relevant to the further analysis that follows. Thus all the later integrals will carry the integration limits θ_{cr} to $\frac{1}{2}\pi$, where θ_{cr} is a known function of Mach number.

3. Acoustic energy flux

The above results plus energy flow relations for a moving medium (Blokhintsev 1946) provide the basis for calculation of the acoustic energy flux per unit area of the shock in terms of the turbulent energy flux and upstream flow Mach number.

For plane unattenuating waves at inclination θ'' (figure 3) the acoustic energy flux propagates in the direction of the 'ray velocity' $\mathbf{V}_S$. The energy flux per unit area is (e.g. Ribner 1958)

$$\mathbf{N}_B = p_1^2 \frac{d\langle p''^2(\theta'')\rangle}{\rho_1 a_1^3} V_F \mathbf{V}_S. \tag{13}$$

Here a_1 is the appropriate sound speed (the value downstream of the shock) and p_1 is the ambient pressure there (note definition of non-dimensional pressure p'' preceding (1)).

The flux of acoustic energy per unit area normal to the shock is the component

$$dI_{\mathrm{ac}}(\theta'') = |N_B| \mathbf{V}_S \cdot \mathbf{U}_1 / V_S U_1. \tag{14}$$

By the geometry of figure 3 this may be written (with $M_1 = U_1/a$)

$$dI_{\mathrm{ac}}(\theta'') = p_1^2 \frac{d\langle p''^2(\theta)\rangle}{\rho_1 a_1} (1 + M_1 \sin\theta'')(M_1 + \sin\theta''). \tag{15}$$

Upon incorporating (12) and integrating over θ'' there results

$$I_{ac} = \frac{3p_1^2\langle u^2\rangle}{2\rho_1 a_1}\int_{\theta_{cr}}^{\frac{1}{2}\pi} |P(\theta)|^2 \cos^3\theta(1+M_1\sin\theta'')(M_1+\sin\theta'')\,d\theta \qquad (16)$$

in terms of an integral over θ. This is the required flux of acoustic energy per unit area normal to the shock.

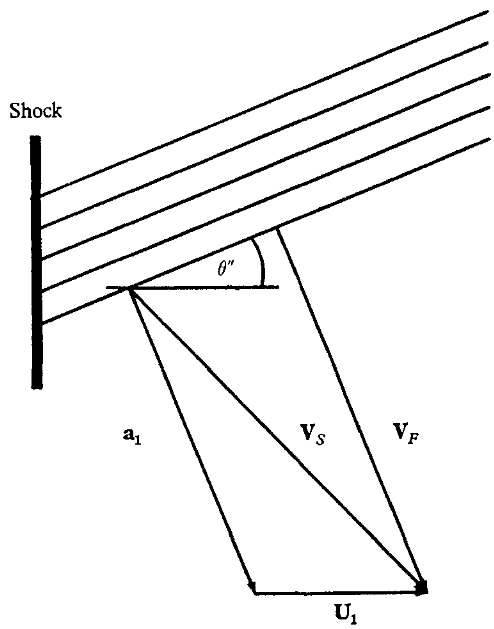

FIGURE 3. Geometry for determining acoustic energy flux in a moving medium.

For comparison the flux of kinetic energy of turbulence into the shock is (recalling the non-dimensional definitions of u, etc., preceding (1))

$$I_{turb} = \langle \tfrac{1}{2}\rho[(U+a^*u)^2+(a^*v)^2+(a^*w)^2](U+a^*u)\rangle - \tfrac{1}{2}\rho U^3 \qquad (17)$$

per unit area. The averaging yields

$$I_{turb} = \tfrac{1}{2}\rho U a^{*2}(3\langle u^2\rangle + \langle v^2\rangle + \langle w^2\rangle) = \tfrac{5}{2}\rho U a^{*2}\langle u^2\rangle, \qquad (18)$$

where the final form results from the postulated isotropy $\langle u^2\rangle = \langle v^2\rangle = \langle w^2\rangle$. The ratio of acoustic to turbulent energy flux is thus, from (16) and (18),

$$\frac{I_{ac}}{I_{turb}} = \frac{3p_1^2}{5\rho\rho_1 a^{*2}a_1 U}\int_{\theta_{cr}}^{\frac{1}{2}\pi} |P(\theta)|^2 \cos^3\theta(1+M_1\sin\theta'')(M_1+\sin\theta'')\,d\theta. \qquad (19)$$

For evaluation of the intensity ratio (19) it is necessary to know the transfer function $P(\theta)$ and the dependence of sound wave inclination θ'' on incident shear wave inclination θ. These quantities are rather cumbersome functions of other functions obtained by Ribner (1953) and evaluated and tabulated (1954) along with the integration limit θ_{cr}. The detailed expressions required in (19) are worked out in the appendix of the present report.

The ratio I_{ac}/I_{turb} has been evaluated for a number of Mach numbers of the flow upstream of the shock. The specific heat ratio is taken as the perfect-gas

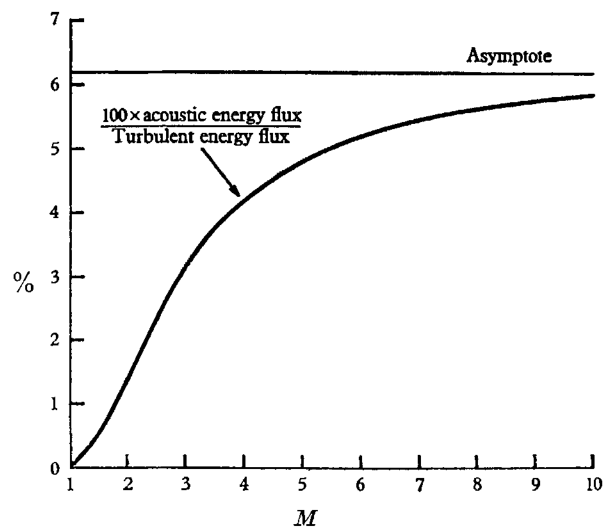

FIGURE 4. Calculated flux of acoustic energy from shock-turbulence interaction. Abscissa scale linear in shock Mach number.

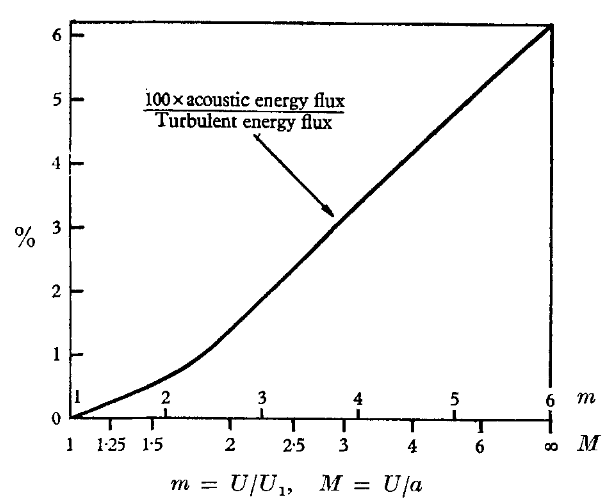

FIGURE 5. Calculated flux of acoustic energy from shock-turbulence interaction. Abscissae scale linear in shock velocity (or density) ratio.

value 1·4. The results are plotted in figures 4 and 5 against the Mach number M and the velocity ratio $m = U/U_1$. In figure 5 the scale is linear in m, which effects a compression of the high Mach number end: the point $M = \infty$ corresponds to $m = 6$.

4. Comparison with Lighthill

Lighthill (1953) has treated the present problem in the limit of very weak shocks. He employed an extension of his general theory of sound generated aerodynamically, with the shock represented by an acoustic step function. The result is expressed as a ratio of 'freely scattered' acoustic energy to the kinetic energy of the turbulence traversed by the shock wave.

Direct comparison with Lighthill's result in its present form is not admissible. His energy relations are reckoned relative to a frame moving with the fluid, whereas ours are reckoned relative to a frame attached to the shock. The fluid-fixed frame is well suited to deal with a patch of quasi-stationary turbulence

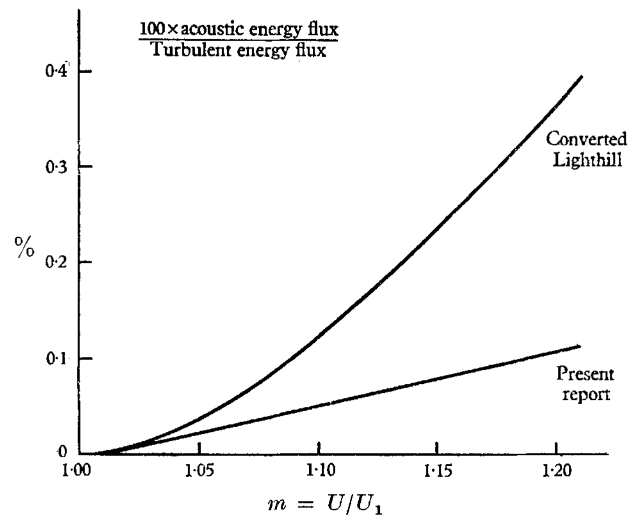

FIGURE 6. Comparison of predictions of Lighthill weak-shock theory (m near unity) and present theory. Lighthill results converted to shock-fixed reference frame.

traversed by a moving shock in a shock tube. The shock-fixed frame, on the other hand, is convenient for supersonic jets in which turbulence passes continuously through a stationary shock pattern. The appropriate expression for acoustic energy flux in the shock-fixed frame, due to Blokhintsev (1946), exhibits conservation of energy for 'ray tubes' passing from the jet to the quiescent air outside.

Conversion of the relevant results of Lighthill to the shock-fixed reference frame is made in appendix B. The final result in the form

$$100\frac{I_{\text{ac}}}{I_{\text{turb}}} = 100\,\frac{\text{flux of acoustic energy}}{\text{flux of turbulent kinetic energy}},\ \%$$

per unit area of shock is designated 'converted Lighthill' as the upper curve of figure 6. The lower curve is the corresponding result of the present report. These are plots of the data of table 1.

The curve of the Lighthill theory (weak shocks) and the curve of the present theory (arbitrary shocks) appear to show a similar asymptotic behaviour for vanishing shock strength; they diverge with increasing shock strength.

Both of these features are to be expected in view of the weak-shock assumption implicit in the Lighthill theory. What is surprising is that the divergence between the two results is already appreciable when m exceeds 1·02. Thus we are inclined to examine the assumptions underlying the two theories in more detail.

In the present theory the shock may have arbitrary strength, but the turbulence velocity is postulated small compared with the (supersonic) flow speed. Ripples or undulations in the shock that develop on passage of the eddies are allowed for. Differences in turbulence intensity across the shock are accounted for; in fact, they are predicted.

M	m U/U_1	$100\ I_{ac}/I_{turb}$ Present report	$100\ I_{ac}/I_{turb}$ Converted Lighthill
1·01	1·017	0·0036%	0·0051%
1·05	1·084	0·0414	0·0905
1·10	1·169	0·0890	0·2817
1·25	1·429	0·252	1·073
1·5	1·862	0·512	—
2·0	2·667	1·39	—
2·5	3·333	2·35	—
3·0	3·857	3·17	—
4·0	4·571	4·20	—
6·0	5·268	5·18	—
∞	6·000	6·20	—

TABLE 1. Acoustic energy flux/turbulence energy flux $= I_{ac}/I_{turb}$

In the Lighthill theory both the shock and the turbulence are weak: the relevant velocities are small compared with the speed of sound. The shock is postulated as planar and moving at constant speed; the rippling motion, which implies a spread of the local shock speeds about a mean, is suppressed. Differences in turbulence intensity across the shock are likewise suppressed.

In both theories the turbulence is treated in effect as a 'frozen' spatial pattern with neglect of the temporal fluctuations.

The assumptions of the Lighthill theory are the more restrictive, not only as to the weak-shock limitation, but otherwise. Suppression of the shock ripples is predicted to have a first-order effect on the sound pressure field. The apparent asymptotic approach of the two theories for $m \to 1$ is therefore not an obvious expectation. The approach is, however, encouraging.

On the other hand, the shortness of the weak-shock region of approximate agreement is very marked in figure 6. Perhaps this is one of those cases—fortunately in the minority—where the range of a first-order theory is very limited indeed.

5. Concluding remarks

Although the generation of sound by shock-turbulence interaction was predicted as early as 1953 (independently by Lighthill, Ribner, and Moore), clear-cut shadowgraph evidence of such sound waves is relatively recent (e.g. the work of Ollerhead 1966). Ollerhead's paper shows the sound waves produced where the turbulence of a supersonic jet passes through the shock pattern. It was just this problem which motivated the present work, namely, the prediction of the intensity of the sound waves emerging from the jet into the outer quiescent air.

However, Lowson (1966) has examined the related problem of surface pressures under the foot of a shock traversed by a turbulent boundary layer. He finds that the near field pressures appear to dominate. This has led him to suggest† that the near-field pressures may be an important factor where shock-turbulence interaction occurs near the edge of a jet: these strong fluctuating pressures imposed on such a pressure-release boundary could greatly enhance the shock-turbulence sound. The present calculations are thus perhaps of poor applicability to the problem: they preclude such enhancement in that they do not allow for a jet boundary near the shock. The near field terms are ignored in our calculation of energy flux, since without an interface they carry zero acoustic energy.

Thanks are due to Drs M. J. Fisher and M. V. Lowson for their interest in the unpublished curves in advance of publication, to Dr Malcolm McChesney for suggestions concerning the exposition of the material, to Mrs Jean Surry for carrying out most of the computations, and to Mr Felix L. Lam for checking the conversion of the Lighthill results.

The work was supported by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR Grant no. 672–66.

Appendix A. Reformulation of equation (19)

We require to express the integrand of (19),

$$\frac{dI_{\mathrm{ac}}/d\theta}{I_{\mathrm{turb}}} = \frac{3p_1^2}{5\rho\rho_1 a_1 a^{*2} U} |P(\theta)|^2 \cos^3\theta (1 + M_1 \sin\theta'')(M_1 + \sin\theta''), \qquad \text{(A 1)}$$

in terms of tabulated functions of θ for purposes of numerical integration.

From Ribner (1954, appendix A)

$$|P(\theta)|^2 = \frac{\gamma^2 m \Pi^2}{\left[\left(\frac{\gamma+1}{2}\right) m - \left(\frac{\gamma-1}{2}\right)\right]^2} \sec^2\theta \sec^2\theta'. \qquad \text{(A 2)}$$

Also
$$\frac{3p_1^2}{5\rho\rho_1 a_1 a^{*2} U} = \frac{3\rho_1^2 a_1^4/\gamma^2}{5\rho\rho_1 a_1 (UU_1) U};$$

since $\rho U = \rho_1 U_1$, $M_1 \equiv U_1/a_1$, $m \equiv U/U_1$ this reduces to

$$\frac{3p_1^2}{5\rho\rho_1 a_1 a^{*2} U} = \frac{3}{5M_1^3 \gamma^2 m}. \qquad \text{(A 3)}$$

† Private communication.

Insertion of (A 2) and (A 3) into (A 1) gives

$$\frac{dI_{ac}/d\theta}{I_{turb}} = \frac{3}{5}\frac{\Pi^2\cos\theta}{\cos^2\theta'}\cdot\frac{(1+M_1\sin\theta'')(M_1+\sin\theta'')}{M_1^3\left[\left(\frac{1+\gamma}{2}\right)m-\left(\frac{\gamma-1}{2}\right)\right]^2} \tag{A 4}$$

as a suitable form of integrand. The functions Π, θ' and θ'' are tabulated against θ in Ribner (1954) for a series of values of M, the flow Mach number upstream of the shock. Values of m corresponding to each M are also listed, and M_1, the Mach number after the shock, can be found from

$$M_1^2 = \frac{(\gamma-1)M^2+2}{2\gamma M^2-(\gamma-1)}, \tag{A 5}$$

or in tables (e.g. Liepmann & Roshko 1957).

Appendix B. Lighthill results referred to moving medium

In Lighthill's (1953) analysis the reference frame and a certain spherical control surface are fixed in the medium behind the shock considered stationary; the shock is in motion. Relative to this frame the acoustic energy crosses unit area of the control surface normal to the propagation vector $\mathbf{a}$ at a rate

$$\text{energy flux} = a^3\frac{(\rho-\rho_0)^2}{\rho_0} = \frac{(p-p_0)^2}{\rho_0 a} \tag{B 1}$$

per unit time.

We wish to go over to a reference frame attached to the shock, so that the medium moves with velocity $\mathbf{U}_1$ relative to the frame. The spherical control surface is unchanged, always containing the same fluid, so it too moves with velocity $\mathbf{U}_1$. On evaluating the energy density in the new frame, acoustic energy crosses unit area of the control surface at a rate (Ribner 1960)

$$\text{energy flux} = \frac{a^3(\rho-\rho_0)^2}{\rho_0}(1+\mathbf{U}_1\,.\,\mathbf{a}/a^2). \tag{B 2}$$

By geometry this is

$$\text{energy flux} = \frac{a^3(\rho-\rho_0)^2}{\rho_0}(1-M_1\cos\theta), \tag{B 3}$$

where Lighthill's θ is the angle between the wave normal and the upstream shock normal.

For air, to the first-order in shock strength $s \equiv \Delta p/p$,

$$M_1 = 1-\tfrac{3}{7}s, \tag{B 4}$$

so that

$$\text{energy flux} = \frac{a^3(\rho-\rho_0)^2}{\rho_0}(1-\cos\theta+\tfrac{3}{7}s\cos\theta). \tag{B 5}$$

On neglecting $\frac{3}{7}s$ compared with unity,

$$\text{energy flux} \approx \frac{a^3(\rho-\rho_0)^2}{\rho_0}\,2\sin^2(\tfrac{1}{2}\theta). \tag{B 6}$$

Accordingly, because of the change of reference frame for evaluating acoustic energy density, Lighthill's equation (60) for total scattered energy acquires a factor $2\sin^2(\frac{1}{2}\theta)$. It follows that the integrand of his equation (73) must be multiplied by the same factor to yield

$$e_{fs} \approx \frac{\epsilon^2}{4}(\tfrac{3}{2}\rho_0\overline{v_1'^2})\int_{\sec^{-1}(1+\alpha)}^{\pi} \cos^2\theta\cos^3(\tfrac{1}{2}\theta)\,2\,d\theta. \tag{B 7}$$

This has the effect of cancelling out the singularity that makes the integral very large in Lighthill's reference frame.

The excluded waves, directed in the range $0 \leqslant \theta \leqslant \sec^{-1}(1+\alpha)$, will overtake the shock and are 'probably mostly absorbed'. The limiting angle $\sec^{-1}(1+\alpha)$ corresponds to the wave angle θ''_{cr} in our notation ($\sec^{-1}(1+\alpha)-\theta''_{cr} = \pi/2$). Thus the waves that are 'probably mostly absorbed' and are therefore excluded correspond to those in our analysis that decay exponentially with distance from the shock. This is the acoustic near field, and is likewise excluded from our integral associated with the far field.

Upon evaluation the integral is

$$e_{fs} = \frac{\epsilon^2}{4}(\tfrac{3}{2}\rho_0\overline{v_1'^2})[1{\cdot}45+4(-\beta+\tfrac{5}{3}\beta^3-\tfrac{8}{5}\beta^5+\tfrac{4}{7}\beta^7)], \tag{B 8}$$
$$\beta \equiv \left(\frac{\alpha}{2(1+\alpha)}\right)^{\frac{1}{2}},$$

where e_{fs} is the total acoustic energy that passes out of the moving control surface when the shock traverses unit volume of turbulent fluid behind the shock. This refers to the shock-fixed reference frame, and thus the flux of acoustic energy is $e_{fs}U_1$ in this frame.

The flux of turbulence kinetic energy into the shock from upstream is given by (7); in Lighthill's (1953) notation this flux is

$$\tfrac{5}{2}\rho_0\overline{v_1'^2}\,U, \tag{B 9}$$

where changes in the turbulence across the shock are neglected. The flux ratio is

$$\frac{I_{ac}}{I_{turb}} = \frac{\text{flux of acoustic energy}}{\text{flux of turbulent energy}} = \frac{3}{20}\frac{U_1}{U}[1{\cdot}45-4\beta+\tfrac{20}{3}\beta^3]\epsilon^2 \tag{B 10}$$

to order β^3.

The various shock-strength parameters are connected as follows for air ($\gamma = \frac{7}{5}$)

$$\epsilon = \frac{U}{U_1}-1 = m-1, \quad s = \frac{p}{p_1}-1 = \tfrac{7}{5}\epsilon, \quad \alpha\,(\text{see Lighthill 1953}) = \tfrac{3}{5}\epsilon, \tag{B 11}$$

to the first-order for weak shocks ($\epsilon \ll 1$). Upon neglecting ϵ compared with unity, U_1/U is replaced by unity in (B 10) to give

$$\frac{I_{ac}}{I_{turb}} = [0{\cdot}2175-0{\cdot}600\beta+\beta^3]\epsilon^2, \qquad \beta = \left(\frac{3\epsilon}{10+6\epsilon}\right)^{\frac{1}{2}},$$
$$= [0{\cdot}1110-0{\cdot}306\beta+0{\cdot}51\beta^3]s^2, \quad \beta = \left(\frac{3s}{14+6s}\right)^{\frac{1}{2}}. \tag{B 12}$$

This is the acoustic/turbulent energy flux ratio as converted from the Lighthill (1953) theory to apply to the present shock-fixed reference frame. It is restricted to weak shocks $\epsilon = (m-1) \ll 1$ by the assumptions of the theory; other assumptions are discussed in the main text.

The comparison of (B 12) with our results computed from (19) herein becomes imprecise except for ϵ so near zero that higher-order terms neglected in (B 11) and elsewhere are truly negligible. That is, (B 12) considered as a function of s will be somewhat different from (B 12) considered as a function of ϵ when the higher-order terms neglected in (B 11) are allowed for. The Lighthill theory, embodying (B 11), makes no distinction.

Added note. Lighthill's final result for the ratio of 'freely scattered' acoustic energy to the kinetic energy of the turbulence $\frac{3}{2}\rho_0\overline{v'^2}$ traversed by the shock wave appears to contain an arithmetical error.

Thus on using $\epsilon = \frac{5}{7}s$, $\alpha = \frac{3}{7}s$, Lighthill's equation (73) leads to

$$0{\cdot}551s^{\frac{3}{2}} - 1{\cdot}054s^2 + 0{\cdot}835s^{\frac{5}{2}}$$

for this ratio, in place of his equation (74),

$$0{\cdot}7s^{\frac{3}{2}} - 1{\cdot}0s^2 + 0{\cdot}7s^{\frac{5}{2}}.$$

This will alter his tabulated values (table 1) which refer to a reference moving with the fluid, rather than our shock-fixed frame.

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